"Exact" Three-Dimensional Linear and Nonlinear Seismic Analysis of Structures with Two-Dimensional Models

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This paper presents a modeling technique by which a complete three-dimensional (3-D) structural analysis of a structure can be performed using two-dimensional (2-D) models, and hence 2-D software. The approach includes the effect of torsion, walls perpendicular and inclined to the direction of motion as well as walls with L, T, and H shapes in plan. Diaphragm displacements are easily modeled. The method can be used with linear and nonlinear analysis. Nonlinearity in the diaphragms can also be modeled with relative ease. Furthermore, unlike the conventional analysis that requires two 2-D models, one in each direction of motion, to model the 3-D structure, this approach requires only a single model. [DOI: 10.1193/1.1623498]

INTRODUCTION

In a traditional two-dimensional (2-D) structural analysis of a three-dimensional (3-D) structure, the seismic resisting elements such as walls and frames in the direction of motion are lined up in a 2-D plane and slaved at the roof and each floor. For ground motion in the perpendicular direction, a completely new 2-D model that includes walls and frames in that direction is required.

Since all seismic resisting elements in a conventional 2-D model undergo equal displacements at each level, the 2-D model represents a 3-D structure in which torsional displacements are locked. This can cause substantial inaccuracies in structures with significant torsion.

Standards and guidelines for design and retrofit of structures typically restrict the use of 2-D analysis when substantial torsion exists. *FEMA-356* (ASCE 2000), for example, defines parameter η, representing torsion, as the ratio of maximum displacement, δ, at any point on the diaphragm to the average displacement at that level ($\delta_{\max}/\delta_{\text{average}}$). If η is less than or equal to 1.5, 2-D analysis is permitted. If η exceeds 1.5, 3-D modeling is mandatory.

Even when 2-D analysis is permitted, amplifying the displacements and forces magnifies the effect of horizontal torsion by factor η for all building elements, even those elements that in the real structure undergo less than average displacement. This requirement, while conservative, may be excessively demanding on more rigid components of a building. Thus components that are adequate may appear to be overstressed in analysis.

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Earthquake Spectra, Volume 19, No. 4, pages 897–912, November 2003; © 2003, Earthquake Engineering Research Institute

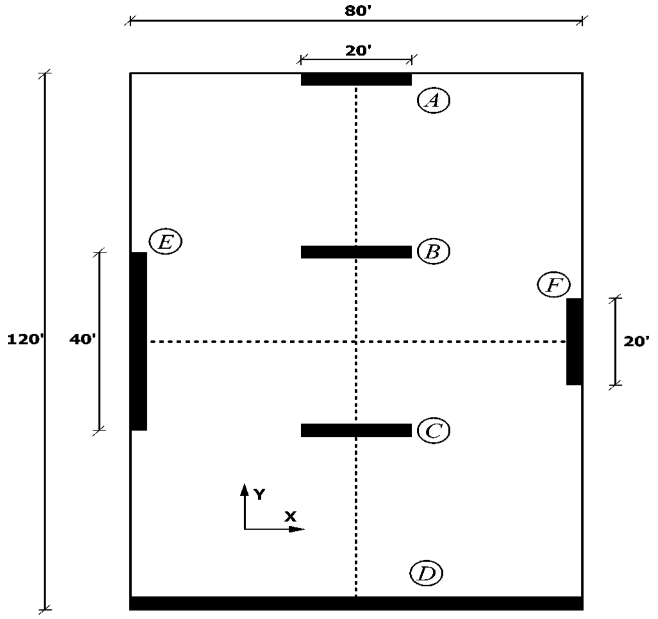

Figure 1. Plan of the example two-story shear wall building.

Therefore, in seismic design guidelines typically either 2-D analysis is not permitted or the more rigid components of the structure are penalized, sometimes to a prohibitive extent.

Other disadvantages of the traditional 2-D analysis include

1. Presence of walls or frames perpendicular to the direction of motion in resisting torsion is ignored.
2. Inclined walls or frames (those not parallel or perpendicular to the direction of motion) cannot be modeled. These walls contribute to resistance in the direction of motion but their inclusion in a conventional 2-D analysis requires much approximation and uncertainty.
3. Intersecting walls and frames such as L-, T-, and H-shaped walls can only be modeled as equivalent sections, and thus the effect of shear lag cannot be easily considered.
4. Two models, one in each direction of motion, are required.
5. Actual and accidental eccentricities cannot be modeled.

The 2-D analysis method presented here will be referred to as "pseudo-3-D." It addresses all the above issues and does so in an "exact" manner compared to a full 3-D analysis. Furthermore, floor diaphragms can be modeled including their deformation and nonlinear performance. It should be noted that the linear 3-D computer programs have presently matured to the point that the proposed pseudo-3-D analysis may not have extensive utility. However, the proposed pseudo-3-D analysis is expected to have signifi-

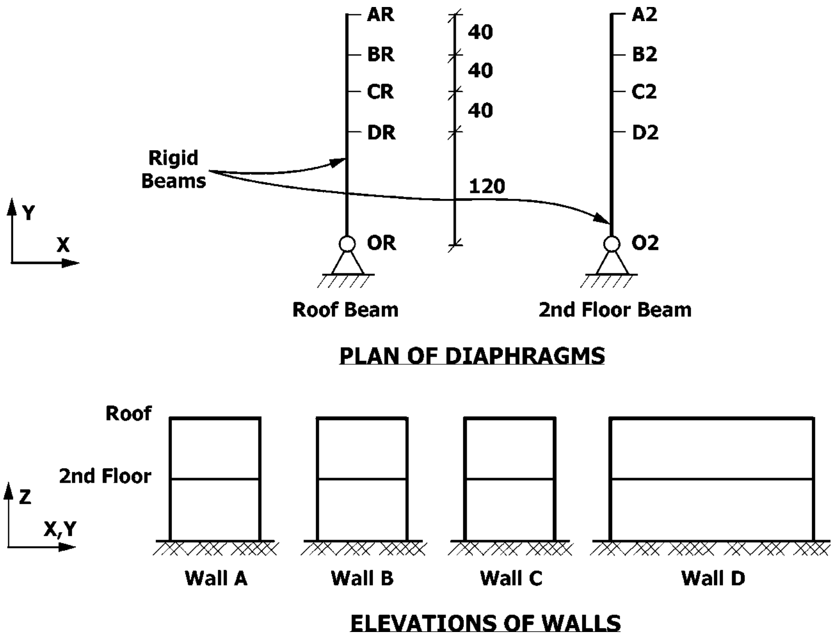

Figure 2. Proposed pseudo-3-D model with predetermined torsion for the building shown in Figure 1.

cant use in nonlinear analysis of structures. In the discussion of the Pseudo-3-D method that follows the seismic resisting elements are assumed, for the purpose of discussion, to be shear walls. The modeling technique, analysis, and conclusions apply equally well for other seismic resisting systems such as braced frames and moment frames.

The development of the proposed method was caused by a practical engineering need to consider eccentricity and torsion in a nonlinear 2-D analysis of a torsionally sensitive structure. Methodologies for considering inclined and intersecting walls were developed later. In this paper we present the methodology using the same chronology as that encountered in its development.

2-D MODEL WITH PREDETERMINED TORSION

To illustrate the concept, consider a two-story structure with a plan view as shown in Figure 1. Wall *D*, due to its longer length, is significantly more rigid than other walls. This 3-D structure, when subjected to forces in the *x* direction would experience torsion causing Wall *A* to displace significantly more than Wall *D*. For the purpose of this discussion let us assume that a linear 3-D analysis shows Wall *A* to deform twice that of Wall *D* at both the roof and the second floor. If a conventional 2-D analysis is used, *FEMA-356* in effect requires Wall *D* to be displaced to the same displacement as Wall *A*. In a nonlinear static analysis this requires pushing Wall *D* to twice the displacement obtained from a 3-D analysis. As indicated above, the development of pseudo 3-D model-

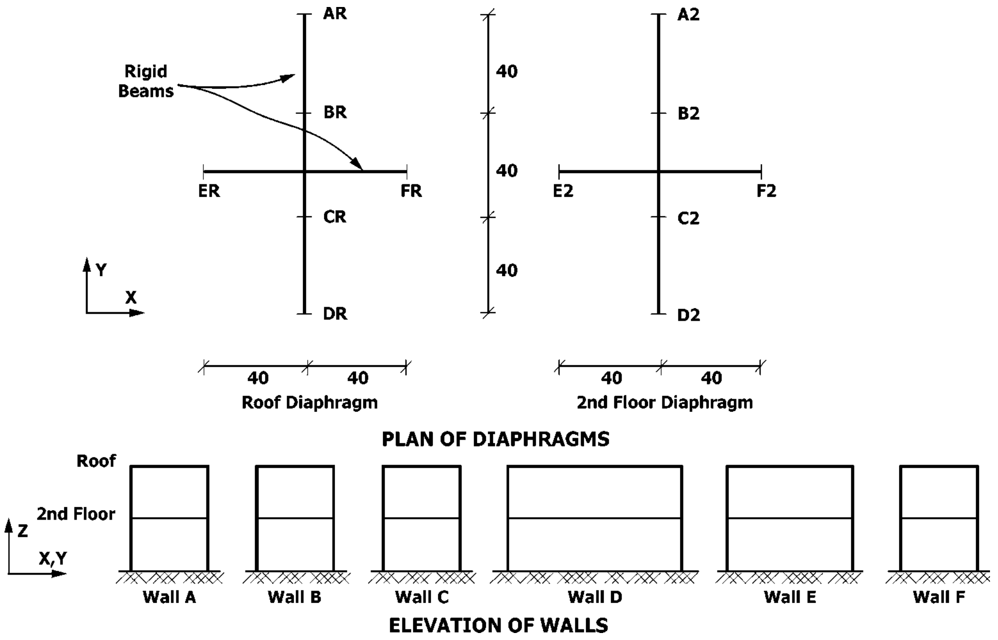

Figure 3. Enhanced pseudo-3-D model for the building shown in Figure 1.

ing was conceived out of a need to perform a pushover analysis of a 2-D model in which Wall *D* automatically is pushed to half the deformation of Wall *A* (the realistic result).

This can be achieved by a 2-D analysis, as follows:

1. Line up walls *A* through *D* in a 2-D model in the conventional way (see Figure 2).
2. Instead of slaving the walls to each other, slave the walls to flexurally rigid beams simply supported at one end and free at the other as shown in Figure 2. One rigid beam is used for slaving at the roof and another rigid beam for slaving at the second floor. One end of these beams is pinned and the other end is free to move in the x direction.
3. All walls and diaphragms shown in Figure 2 are included in a single 2-D analytical model.

The slaving pattern is as follows:

- Slave the x displacement of Wall *A* at the roof to the x displacement of point *AR* of roof rigid beam.
- Similarly, slave x displacement of walls *B*, *C*, and *D* at roof to the x displacement of points of *BR*, *CR*, and *DR* of the roof rigid beam.
- Finally, slave x displacement of walls *A*, *B*, *C*, and *D* at the second floor to the x displacement of points *A*2, *B*2, *C*2, and *D*2 of the second-floor rigid beam.

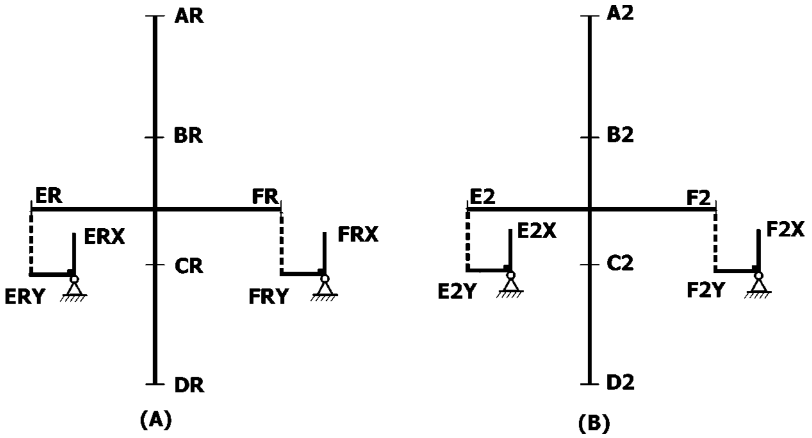

Figure 4. Modeling horizontal diaphragms with compatibility angles: (a) roof diaphragm, and (b) second floor diaphragm.

Note that since distance *AR*-*OR* is twice that of *DR*-*OR*, Wall *A* at the roof will always have twice the displacement of Wall *D* at the roof. The same relationship applies at the second floor. Similarly, in order for walls *B* and *C* to be forced to the same displacement as they would in a 3-D model, we select points *BR* and *CR* within the line segment *AR*-*DR* proportional to the location of walls *B* and *C* in the actual structure. Points *B*2 and *C*2 are also similarly selected according to the wall locations. In fact, the roof rigid beam and second floor rigid beams are substitutes for the roof and second-floor diaphragms.

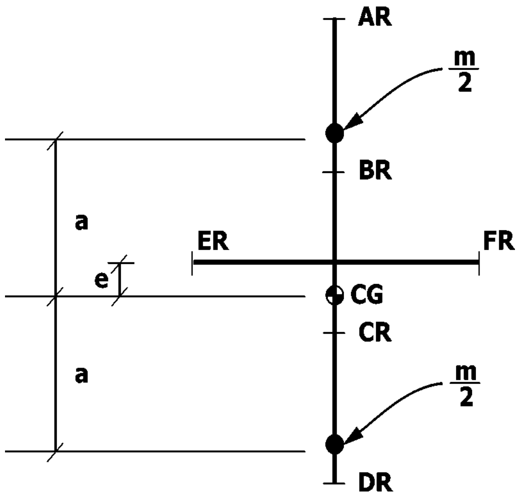

Figure 5. Modeling of roof diaphragm mass.

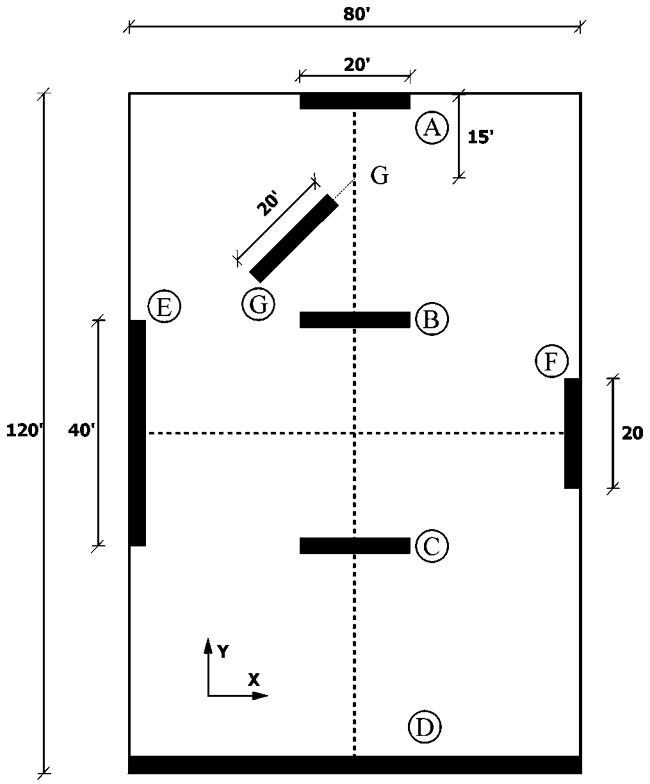

Figure 6. Building with an added inclined wall.

A pushover analysis of the model indicated above would subject all walls to displacements proportional to displacements of the walls obtained from a full 3-D linear analysis.

ENHANCED 2-D TORSION MODEL

The model discussed above considers torsion of the 3-D structure but requires a previous knowledge of the relative displacements of different walls. Furthermore, in an actual nonlinear building, yielding of walls causes change in the relative stiffness of walls and thus the location of points *OR* and *O*2 in Figure 2 will change during response to ground motion. To accommodate this behavior, we can remove the restraints at *OR* and *O*2 and allow the diaphragm rigid beams to displace as dictated by their compatibility and equilibrium with the connected walls. Note, however, that in the restrained model of the previous section, the wall displacements follow the displacement of a 3-D model in which contribution of walls *E* and *F* in the *y* direction is included. Therefore, in the enhanced 2-D model, removal of the constraint at *OR* and *O*2 needs to be accompanied by introducing the effect of walls *E* and *F* in the 3-D model to resist torsion.

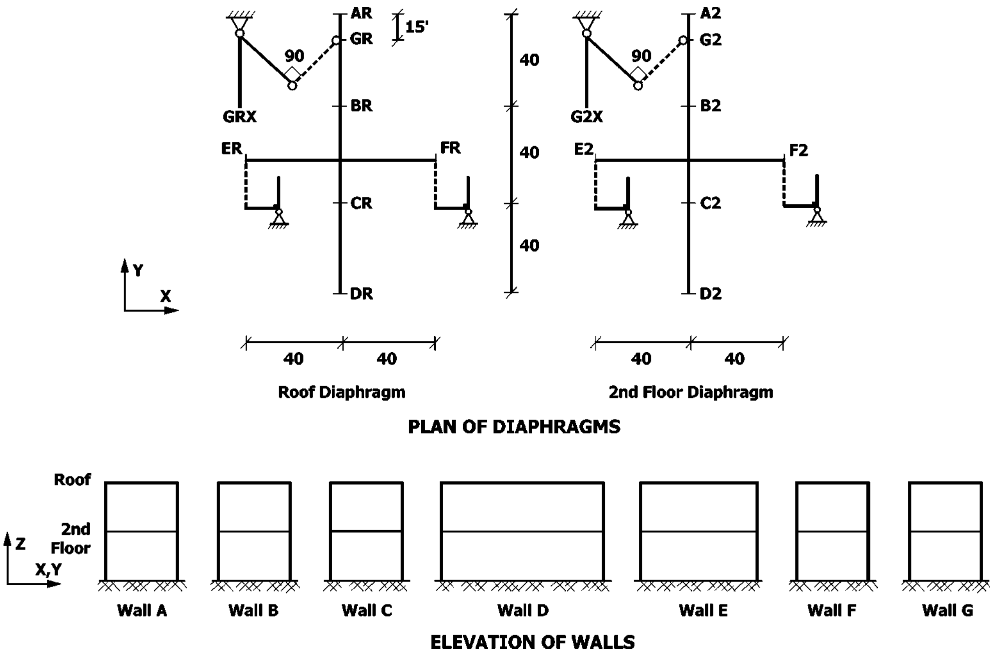

Figure 7. Pseudo-3-D model for the building with an inclined wall.

This can be achieved by adding walls *E* and *F* to the wall line-up and adding a horizontal rigid beam to each of the previous vertical rigid beams so each diaphragm rigid-beam system now consists of a cross as shown in Figure 3. Similar to the selection of other points on the rigid diaphragm, points *ER* and *FR* are also selected proportional to the actual location of the walls *E* and *F*.

For walls in the *y* direction:

- Slave *x* displacement of Wall *E* at the roof to *y* displacement of point *ER* of the roof diaphragm.
- Similarly, slave the *x* displacement of Wall *E* at the second floor to the *y* displacement of point *E*2 of the second floor diaphragm.
- Finally, slave the *x* displacement of Wall *F* at the roof and second floor to the *y* displacement of points *FR* and *F*2, respectively.

The *y*-axis of the diaphragm system in our 2-D model represents the third dimension of the 3-D structure, and our single 2-D model can analyze the building in both directions of ground motion simultaneously, exactly the same as in a full 3-D model. The lateral loads can be applied at the desired locations on the diaphragm in the same way it is done in the 3-D analysis. Therefore, both actual and accidental eccentricities can now be accurately modeled.

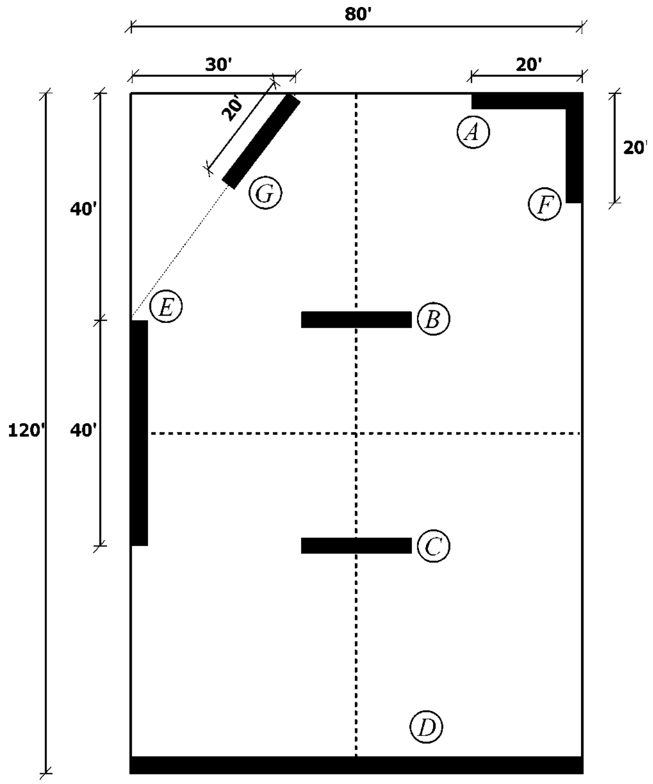

Figure 8. Three-story example structure.

SOME INTERESTING OBSERVATIONS

The following observations regarding the enhanced 2-D torsion model are of particular interest:

1. The roof and/or second-floor diaphragm need not be rigid. Instead, the beams of these elements can have the appropriate shear and flexural rigidity and strength to model diaphragm displacement and nonlinearity. Therefore, buildings with a combination of rigid and flexible floors and roofs can be easily modeled.
2. The roof and floor diaphragms do not necessarily need to be limited to beam elements as shown in Figure 3. The diaphragm modeling may be comprised of shell elements having the shape of the building diaphragms complete with chords, drags, and openings. Thus diaphragms can be modeled with the same level of detail as the vertical seismic-resisting elements with relative ease.
3. Some 2-D software may not allow slaving of y displacement of one node to the x displacement of another node. This is only an inconvenience but can be overcome by introducing for each such diaphragm node, a "compatibility angle" as shown in Figure 4. A "compatibility angle" consists of two rigid beam elements rigidly connected to each other and supported by a pin at their point of connec-

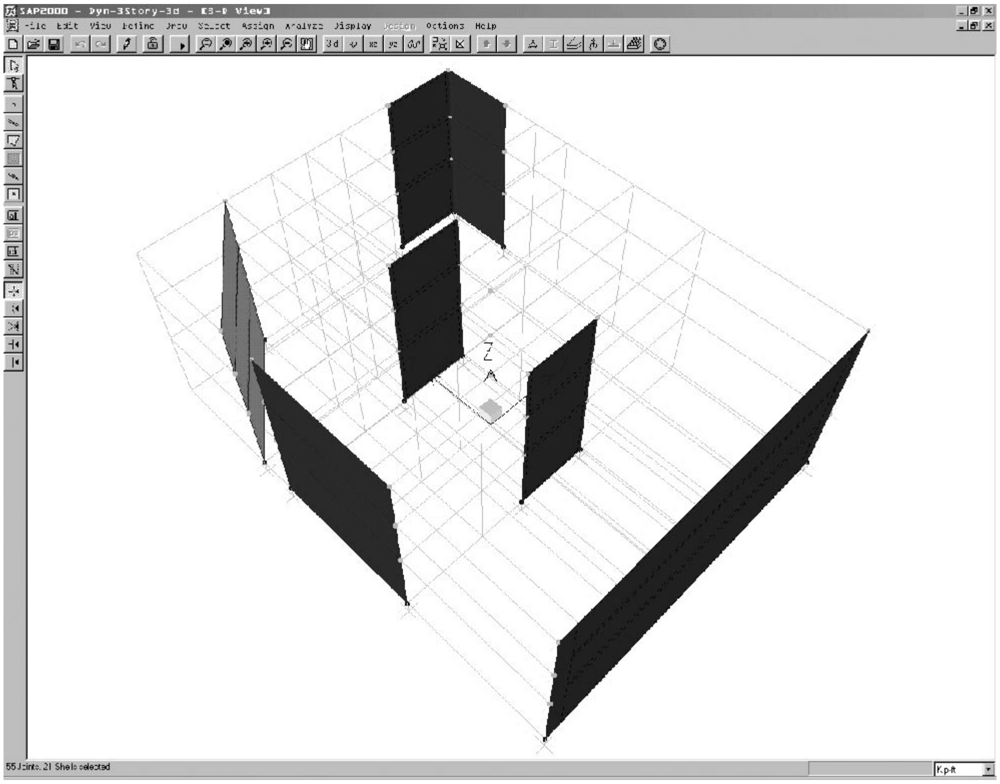

Figure 9. The full 3-D model of the three-story example structure.

tion. Since the end displacements of the two members in this mechanism are equal, the "compatibility angle" converts an x displacement into an equal y displacement. The slaving for compatibility angle is as follows:

Slave the x displacement of Wall E at the roof to x displacement of point ERX on the compatibility angle.

Slave y displacement of point ER on the diaphragm to y displacement of point ERY on the compatibility angle.

Similar compatibility angles and slaving are used for points FR, $E2$, and $F2$.

4. For dynamic analysis, roof and floor masses can be located on the diaphragm as desired to consider eccentricity. Torsional modes of vibration can be captured by including rotational mass moment of inertia on the diaphragm model. Alternatively, the mass at each floor may be represented by two mass points, as shown in Figure 5, each having one half of the total mass of the floor with center of gravity at the desired eccentricity. The masses would be located at a distance a from the center of gravity of the system so that

$$ma^2 = \text{mass moment of inertia}$$

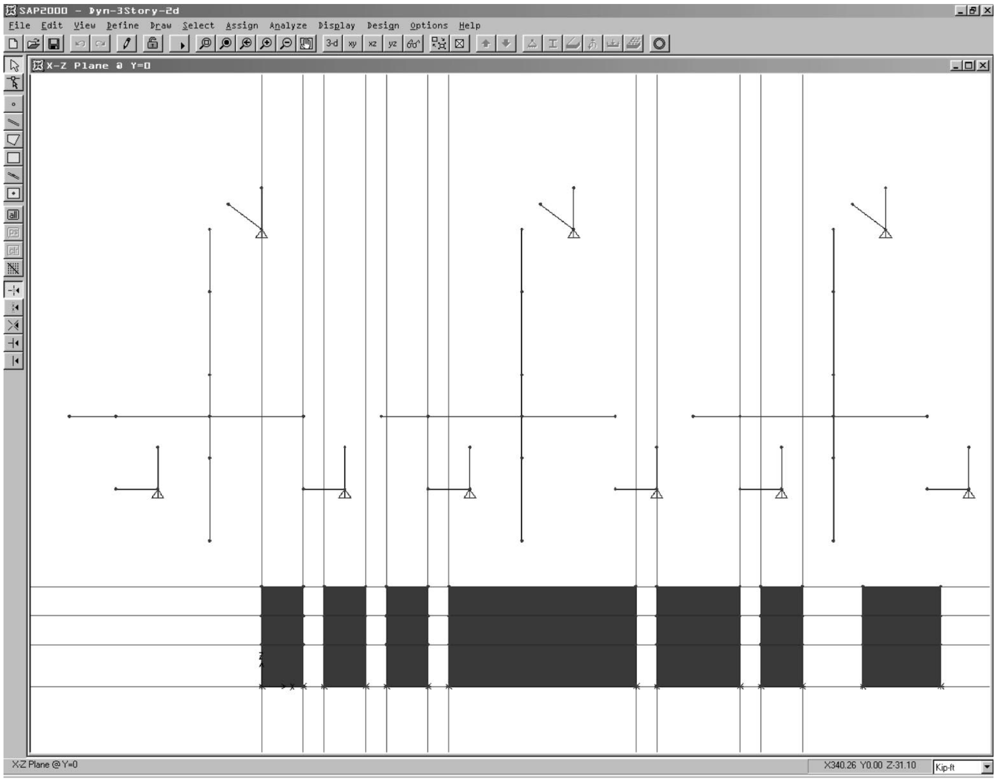

Figure 10. The pseudo-3-D model of the three-story example structure.

5. Note that mode shapes and frequencies obtained from the 2-D model include both translational (*x* and *y* directions) and torsional modes of the 3-D structure. Thus all important components of the full 3-D structure are included in the model. The out-of-plane motion of walls and diaphragms, however, are efficiently excluded.

THE COMPLETE PSEUDO-3-D MODEL

Two additional enhancements explained below make the proposed pseudo-3-D model just as powerful as a full 3-D model.

INCLINED WALLS

Vertical elements resisting lateral loads with their plane neither in *x* nor in *y* direction can also be modeled in an approach similar to modeling of y walls with compatibility angles. Wall *G*, as shown in Figure 6, would be lined up with other walls. Its roof and second floor would be connected to compatibility angles as shown in Figure 7 and described below.

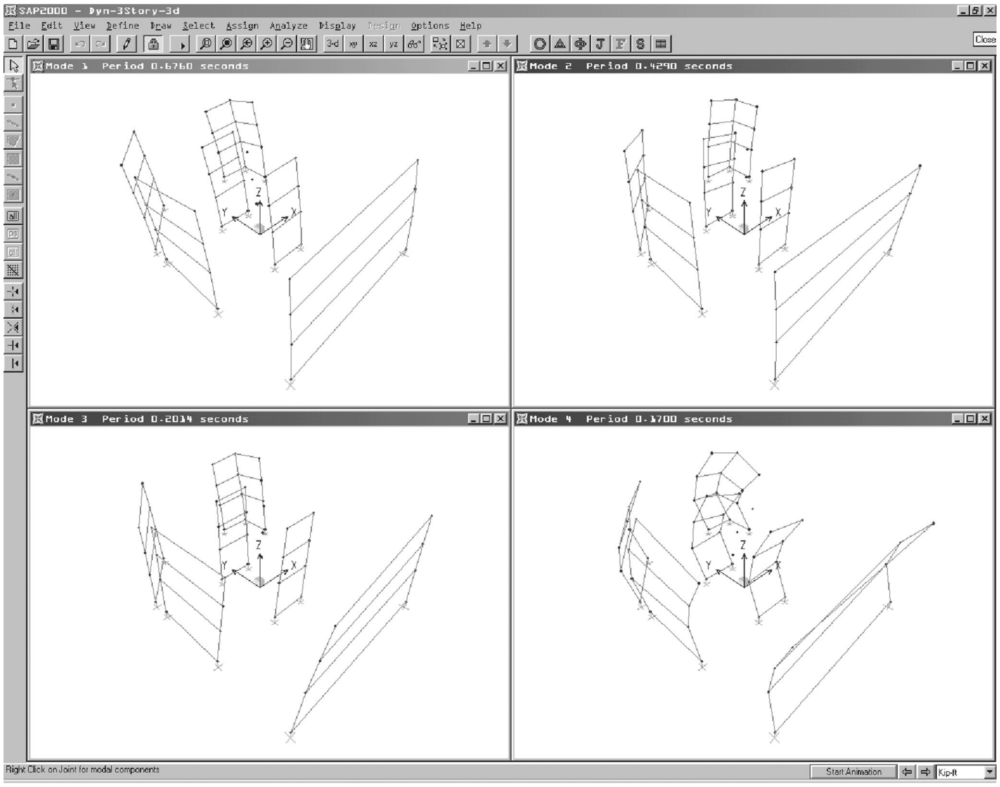

Figure 11. The first four modes of vibration of the full 3-D model.

- The compatibility angle has one leg vertical and the other leg perpendicular to wall *G*.
- The *x* displacement of Wall *G* at the roof is slaved to the *x* displacement of point *GRX*.
- Point *GR* on the diaphragm model is slaved, for motion parallel to the wall, to the other end of the compatibility angle (or connected with a pin-ended rod along the wall axis as shown in Figure 7).

FLANGED WALLS

Flanged walls have two structurally significant effects:

1. Each wall in *x* or *y* direction has its in-plane stiffness and strength
2. Walls in the *x* direction act as flange for walls in the *y* direction and vice versa, thus providing composite action with the associated increase in stiffness and strength. Modeling in 3-D will provide the appropriate shear lag in this composite behavior.

Both of the above effects can be captured in a pseudo-3-D analysis. This is achieved

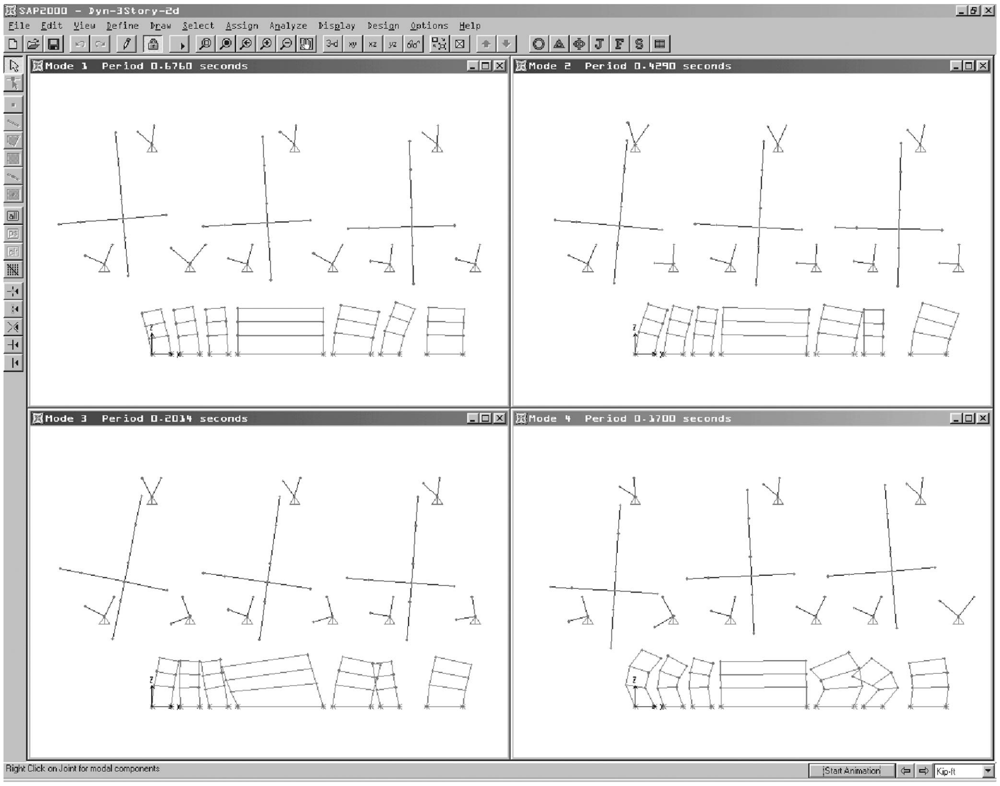

Figure 12. The first four modes of vibration of the pseudo 3-D model.

by slaving the vertical displacement of the nodes at the intersection of walls. This provides exactly the same composite action and shear lag behavior as in the full 3-D analysis, as shown in the following examples.

EXAMPLE PROBLEMS

The following examples help to illustrate the use and accuracy of the proposed modeling technique.

EXAMPLE 1

In order to check that the pseudo-3-D analysis described above accurately represents the full 3-D structure, a three-story structure, as shown in Figure 8, was analyzed both in 2-D and 3-D. Both static and dynamic analyses were performed. The SAP2000 computer program (Computers and Structures 2002) was used for a full 3-D analysis. The pseudo-3-D analysis also used SAP2000 using the modeling techniques described above. Figures 9 and 10 show the full 3-D and the pseudo-3-D models, respectively.

Note that all walls, whether in the *x* direction, *y* direction, or inclined, are included in the 2-D model. Because the structure has three stories, there are three diaphragm struc-

Table 1. Comparison of results obtained from full-3-D and pseudo-3-D models

Response Parameter	Pseudo-3-D Analysis	Full-3-D Analysis
Static Analysis Displacements*		
Wall A	0.1678	0.1678
Wall B	0.1246	0.1246
Wall C	0.0813	0.0813
Wall D	0.0381	0.0381
Wall E	0.0023	0.0023
Static Analysis Forces**		
Wall A	6,188	6,188
Wall B	3,520	3,520
Wall C	2,656	2,656
Wall D	14,201	14,201
Wall E	574	574
Wall F	2,862	2,862
Vibration Periods (sec.)		
Mode 1	0.6760	0.6760
Mode 2	0.4290	0.4290
Mode 3	0.2014	0.2014
Mode 4	0.1700	0.1700
Mode 5	0.1070	0.1070
Mode 6	0.0906	0.0906

* Values are in feet at the roof level, in plane of the wall, for 10,000 kips applied at the center of mass of roof and each floor.

** Values are in kips for shear at the base, in plane of the wall, for 10,000 kips applied at the center of mass of roof and each floor.

tures. Also note compatibility angles. For this analysis, the diaphragms were assumed rigid in the full 3-D analysis. Thus in the equivalent pseudo 3-D model, the mechanical and section properties (E and I and A) of the diaphragm elements were set to be very high. Figures 11 and 12 shows the first four modes of vibration for the full 3-D and the pseudo-3-D models.

The comparison of the full 3-D and pseudo-3-D analysis is shown in Table 1. Results show that all output parameters including forces, displacements, periods, and mode shapes are identical.

EXAMPLE 2

Structures with nonrectangular diaphragms can be modeled in a similar fashion. The structure with walls as indicated in Figure 13 cannot be reliably modeled using conventional 2-D modeling.

The pseudo-3-D model of the structure, however, is quite possible and practical, as shown in Figure 14.

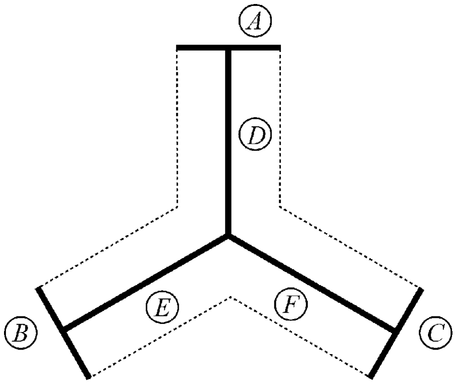

Figure 13. Plan of building example No. 2.

Although the example structure has two stories, for clarity only one diaphragm level (roof) is shown here. Note that nodes 20 through 24 are on the vertical leg of compatibility angles to be slaved to walls. The other legs of compatibility angles are perpendicular to the corresponding walls and are connected to the diaphragm by a pin-ended rod oriented parallel to the wall. The slaving of roof nodes is as follows:

For connection of walls to diaphragms:

Wall D: $11x \Leftrightarrow 20x$ (i.e., slave the x displacement of nodes 11 and 20)

Walls E and F: $13x \Leftrightarrow 21x$, $15x \Leftrightarrow 22x$

Walls A, B, and C: $2x \Leftrightarrow 16x$, $5x \Leftrightarrow 23x$, $8x \Leftrightarrow 24x$

And for composite action of walls:

Walls D, E, and F: $10y \Leftrightarrow 12y \Leftrightarrow 14y$,

Walls A, B, and C: $2y \Leftrightarrow 11y$, $5y \Leftrightarrow 13y$, $8y \Leftrightarrow 15y$

CONCLUSIONS

A novel method for "exact" analysis of 3-D structures using a 2-D modeling environment is presented. Presently, there are several computer programs capable of full-3-D linear and nonlinear analysis of structures. Large 3-D building models, however, often contain thousands of degrees of freedom. Consequently, full-3-D nonlinear time-history analysis of such structures may require a prohibitive amount of computing time. The proposed modeling technique has the following advantages:

1. *Efficiency*. For seismic analysis, the out-of-plane motion of diaphragms and walls are typically not used and are thus wasted degrees of freedom. The pseudo-3-D modeling has significantly fewer degrees of freedom and, therefore, is much more efficient to solve.
2. *Simplicity*. Modeling of floor diaphragms as beams rather than shell elements simplifies consideration of diaphragm displacement and nonlinearity. The de-

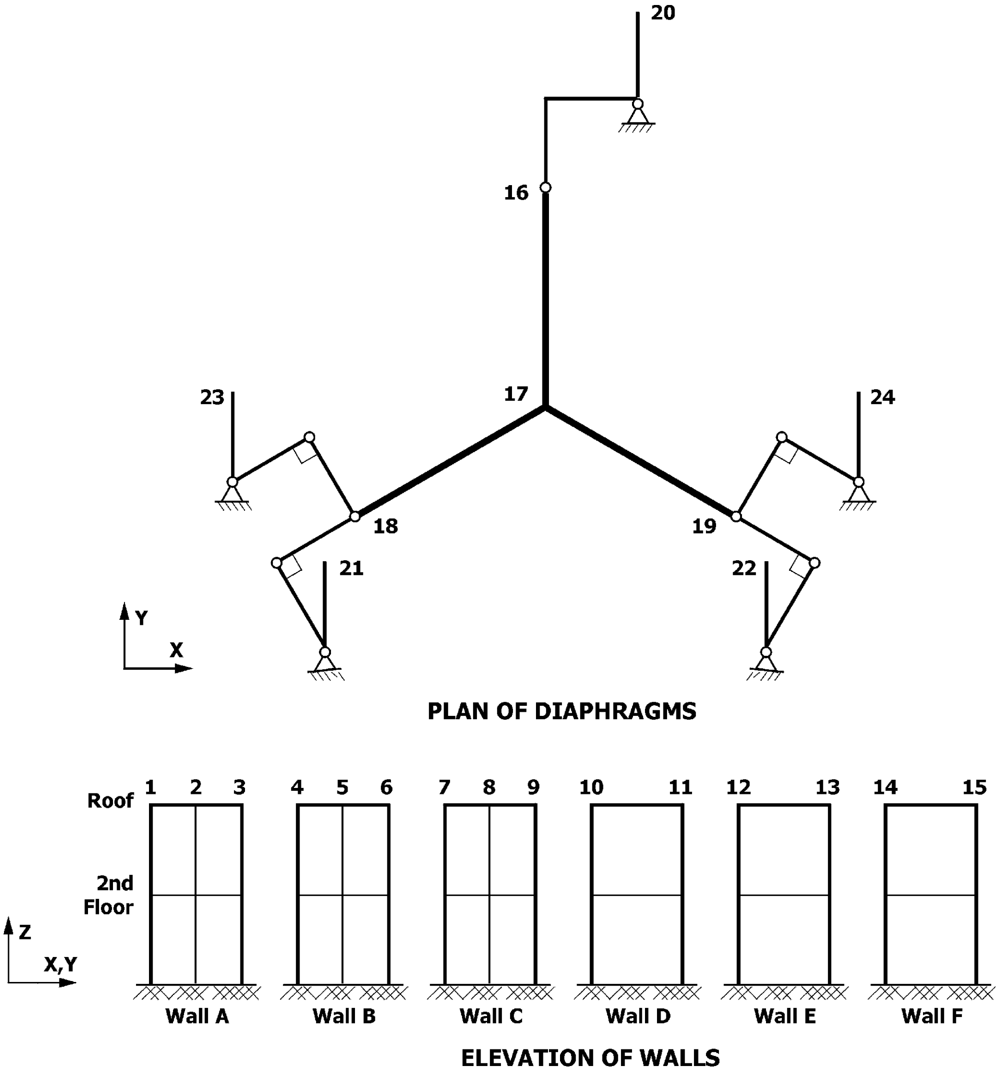

Figure 14. Pseudo-3-D model of building example No. 2.

sign and/or evaluation of diaphragm are also simplified because the output is total shears and moments that are the parameters of particular interest to an engineer. The engineer is relieved of the tedious task of tracking and integrating the shearing and axial stresses in the shell finite elements of a full 3-D model.

3. *Presentation*. The presentation of results is both more efficient and simpler to understand. All elements of interest, i.e., walls in the x and y direction, and diaphragms of all levels are presented in one plane, all at one time. A color plot of demand capacity ratios shows the in-plane behavior of all elements of the structure on a single screen.

4. *Availability*. The use of pseudo-3-D modeling permits the use of available and much more affordable 2-D software for 3-D analysis.
5. *Accuracy*. Since this model represents exactly the actual 3-D structure, the penalty factors provided in *FEMA-356* and similar seismic standards for 2-D analysis need not be imposed.

The proposed approach, however, considers only in-plane behavior of individual or intersecting plane frames, shear walls, and braced frames. As a result, it cannot model biaxial behavior of members. For example, it cannot model the biaxial nonlinear behavior a corner column. Concurrent biaxial ground motion excitations, however, can be readily modeled.

The examples presented in this paper use linear analysis for simplicity. The proposed method, however, satisfies equilibrium and compatibility of all vertical members as well as floor and roof diaphragms. Therefore, changes in strength and stiffness that may occur during nonlinear response are automatically included.

The approach presented in this paper can be used to include diaphragm flexibility using 2-D software. Its greatest strength, however, is its application in conjunction with 2-D nonlinear analysis programs in order to accurately model 3-D structures. In addition, the techniques presented can be incorporated into existing 2-D computer programs to automatically include diaphragm elements, “compatibility angles,” and slaving constraints.

ACKNOWLEDGMENTS

Assistance of Mr. Mohamed Al Satari with development of SAP2000 computer models utilized in this study is gratefully appreciated.

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(Received 17 October 2002; accepted 29 April 2003)